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Effects of thermal radiation in a thermocapillarity thin film flow on an unsteady stretching sheet

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Abstract

This paper examined the effects of thermocapillarity and thermal radiation on the boundary layer flow and heat transfer in a thin film on an unsteady stretching sheet. The governing partial differential equations are reduced to ordinary differential equations by a similarity transformation and hence solved by using Homotopy Analysis Method. The effect of thermal radiation is considered in the energy equation and the variations of dimensionless surface temperature as well as the heat transfer characteristics with various values of Prandtl number, thermocapillarity number, radiation parameter are graphed and tabulated.

Keywords: Thin film; stretching sheet; thermocapillarity; thermal radiation; Homotopy Analysis Method (HAM).

1. Introduction

Boundary layer flow and heat transfer in a thin liquid film on an unsteady stretching sheet with thermal radiation is important in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry and other industrial engineering processes. Wang [1] was the pioneer in investigating the hydrodynamics of a flow in a thin liquid film on an unsteady stretching surface. Later, Andersson et al. [2] studied the heat transfer characteristics of the hydrodynamical problem solved by Wang[1]. Liu and Andersson [3] examined the problem with a more general form of prescribed temperature variation of a stretching sheet.

Wang [4] investigated the same problem of Andersson et al. [2], presenting analytic solutions. Several researchers extended Wang's [1] classical problem taking into consideration in non- Newtonian [5, 6, 7, 8, 9], thermocapillarity [10, 11, 12, 13] and magnetic effects [14, 15]. The aim of this investigation is to extend the model in Wang [4] by taking into account the thermocapillarity and thermal radiation effects and employ the similarity transformation introduced by Wang [4] to transform the extent of the independent variable into a finite range 0–1. The solutions reached using HAM are presented and implications discussed.

2. Problem Statement

The fluid flow modeled as an unsteady, two-dimensional, incompressible viscous laminar flow on a horizontal thin elastic sheet, emerges from a narrow slot at the origin of a Cartesian coordinate system. The model is shown schematically in Fig. 1.

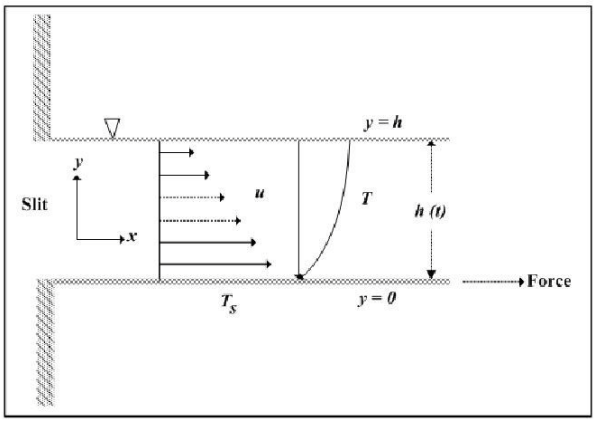

Figure 1: Schematic diagram of thin film flow on an unsteady stretching sheet

The fluid motion and heat transfer arise in the stretching of the horizontal elastic sheet. It is assumed that the sheet temperature varies with the coordinate x and time t. Under these assumptions, the governing conservation equations of mass, momentum and energy at unsteady state can be expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\rho C_P\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}, \tag{3}$$

subject to

$$u=U,\quad v=0,\quad T=T_s \quad \text{at} \quad y=0, \tag{4}$$

$$\mu\frac{\partial u}{\partial y}=\frac{\partial \sigma}{\partial x},\qquad \frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=0,\quad v=\frac{dh}{dt} \quad \text{at} \quad y=h, \tag{5}$$

where u and v are the velocity components of the fluid in the x- and y- directions, T is the temperature, ν is the kinematic viscosity, ρ is the density, C_P is the specific heat at constant pressure, κ is the thermal diffusivity and $h(t)$ is the thickness of the liquid film. The velocity of the stretching surface is defined as $U=bx/(1-\alpha t)$, with α and b as positive constants. The elastic sheet's temperature is assumed to vary both along the sheet and with time in accordance with

$$T_s=T_o-T_{ref}\frac{\partial x^2}{\nu}(1-\alpha t)^{-\frac{3}{2}}, \tag{6}$$

where T_o is the temperature at the slit, T_{ref} is the constant reference temperature for all $t<1/\alpha$, d is the positive constant of proportionality with dimension $(\text{length}^{2-r}\,\text{time}^{-1})$.The surface of the planar liquid film is assumed to be smooth and free of surface waves, while viscous shear stress and heat flux are assumed to vanish at the adiabatic free surface. The radiative heat flux q_r under Rosseland approximation (Brewster, 1992, cited in [12]) is given in the form

$$q_r = -\frac{4\sigma}{3k_1}\frac{\partial T^4}{\partial y}, \tag{7}$$

where σ is the Stefan Boltzmann constant and k_1 is the mean absorption coefficient. We assume that the temperature difference within the flow is sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_o and neglecting higher-order terms, thus:

$$T^4 \cong 4T_o{}^3 T - 3T_o{}^4 . \tag{8}$$

2.1. Similarity Transformations

By using the similarity transformations developed by Wang [1], Eq. (1) - Eq. (5) are transformed to the following nonlinear boundary value problem:

$$f''' + \gamma\left(ff'' - \frac{1}{2}S\eta f'' - (f')^2 - Sf' \right) = 0, \tag{9}$$

$$\frac{1}{\Pr}(1+N_R)\theta'' + \gamma\left(f\theta' - 2f'\theta - \frac{1}{2}S\eta\theta' - \frac{3}{2}S\theta \right) = 0, \tag{10}$$

subject to

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \tag{11}$$

$$f(1) = \frac{1}{2}S, \quad f''(1) = M\theta(1), \quad \theta'(1) = 0, \tag{12}$$

where N_R is the radiation parameter defined as $N_R = 16\sigma_* T_o{}^3 / \kappa k_1$ and the thermocapillarity number M is given as

$$M = \frac{2\beta\delta\sigma_0 T_{\text{ref}}}{\mu\sqrt{b^3\nu}}. \tag{13}$$

The most important characteristics of flow and heat transfer are the shear stress τ_s and the heat flux q_s on the stretching sheet that are defined as

$$\tau_s = \mu\left(\frac{\partial\mu}{\partial y}\right)_{y=0}, \tag{14}$$

$$q_s = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{15}$$

where μ is the fluid dynamic viscosity. The local skin-friction coefficient C_f and the local Nusselt number Nu_x can be defined as

$$C_f = \frac{\tau_s}{\rho u^2/2}, \tag{16}$$

$$Nu_x = -\frac{x q_s}{\kappa T_{\text{ref}}}. \tag{17}$$

Thus, the skin friction and the rate of heat transfer for fluid flow in a thin film can be expressed as

$$\frac{1}{2}C_f \operatorname{Re}_x^{1/2} = \frac{1}{\beta} f''(0), \tag{18}$$

$$Nu_x \operatorname{Re}_x^{-1/2} = \frac{dx^2}{\beta\nu(1-\alpha t)^{\frac{1}{2}}}\theta'(0), \tag{19}$$

where $\operatorname{Re}_x = Ux/\nu$ is the local Reynolds number.

3. Solution Approach

We employed HAM to solve Eq. (9) – Eq. (12) with the aid of Maple. The Homotopy Analysis Method (HAM) is a general analytic method to get series solutions of various types of nonlinear equations [14, 15]. Several studies have successfully applied HAM to various nonlinear problems in science and engineering.

4. Results & Discussion

The effects of radiation parameter and thermocapillarity on the film thickness β, surface shear stress $f''(0)$, free surface temperature $\theta(1)$ and heat flux $-\theta'(0)$ at a specific thermocapillarity number M are shown in Table 1 and Table 2, respectively. It is concluded that the film thickness and the free surface temperature increases as the radiation parameter increases and the surface shear stress and dimensionless heat flux are decreases. By increasing the thermocapillarity number, it will increase the film thickness and dimensionless heat flux and decrease the surface shear stress and temperature distribution at a fixed value of $N_R = 1$.

Thermal radiation plays a significant role in controlling the temperature of the thin film flow. The effects of the different values of radiation parameter N_R on the temperature profiles are depicted in Figure 1. By fixing the thermocapillarity number M = 1 and unsteadiness parameter S = 1.4, it is observed that temperature distribution of the thin film flow increases as N_R increases. Figure 2 shows that by increasing the value of the thermocapillarity number causes a decline in both velocity and temperature profiles.

Table 1: Variations of $\gamma(\eta)$, β, $f''(0)$, $\theta(1)$ and $-\theta'(0)$ using 10th-order HAM approximation when $\Pr = 1$, $M = 1$ and N_R is varied.

N_R	$\gamma(\eta)$	β	$f''(0)$	$\theta(1)$	$-\theta'(0)$
0	4.957481	2.226540	-2.778245	0.096298	3.730108
0.5	5.171979	2.274198	-2.811444	0.136355	3.045699
1.0	5.444928	2.333437	-2.876091	0.196980	2.639235
1.5	5.645566	2.376040	-2.950143	0.265046	2.328770
2.0	5.723277	2.392337	-3.003711	0.318547	2.070134

Table 2: Variations of $\gamma(\eta)$, β, $f''(0)$, $\theta(1)$ and $-\theta'(0)$ using 10th-order HAM approximation when $\Pr = 1$, $N_R = 1$ and M is varied.

M	$\gamma(\eta)$	β	$f''(0)$	$\theta(1)$	$-\theta'(0)$
0	4.631078	2.151994	-2.680943	0.390103	2.149368
0.5	5.054291	2.248175	-2.796270	0.223472	2.518114
1.0	5.444928	2.333437	-2.876091	0.196980	2.639236
1.5	5.678298	2.382918	-2.928594	0.175077	2.713842
2.0	5.952701	2.439816	-3.012912	0.176705	2.760849

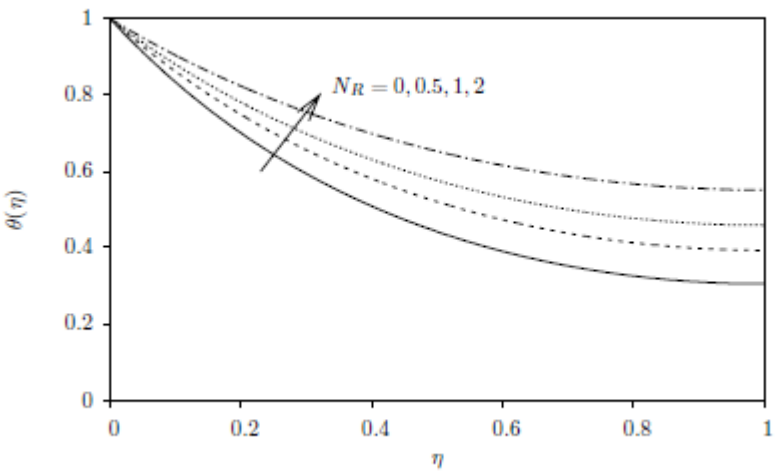

Figure 1: The effects of N_R on the temperature distributions $\theta(\eta)$ using 10th-order HAM approximation for the case of $h_f = -0.6$, $h_\theta = -0.8$, $S = 1.4$, $\Pr = 1$, $M = 1$.

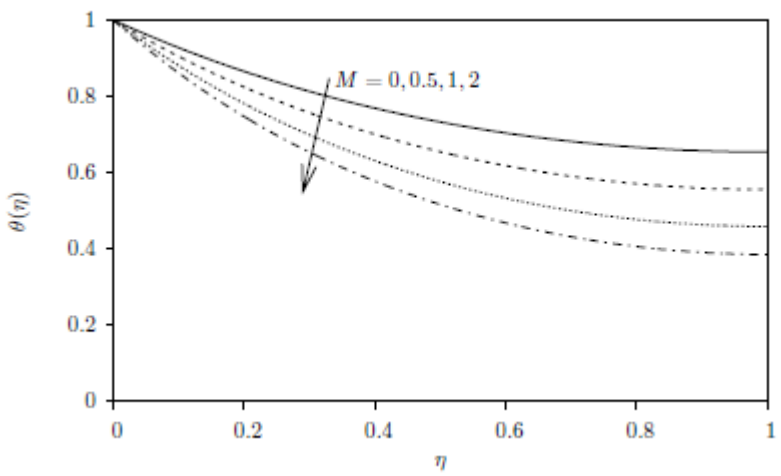

Figure 2: The effects of M on the temperature distributions $\theta(\eta)$ using 10th-order HAM approximation for the case of $h_f = -0.6$, $h_\theta = -0.8$, $S = 1.4$, $\Pr = 1$, $N_R = 1$.

5. Concluding Remarks

The effects of thermal radiation in a thermocapillarity thin liquid film on an unsteady stretching sheet was analyzed successfully by means of the homotopy analysis method (HAM) in this paper. For a specified values of *S*, Pr and *M*, decreasing radiation parameter is found to thicken the boundary layer thickness, hence increase the temperature distributions and decrease the dimensionless heat flux.

Acknowledgment

The authors would like to acknowledge the financial support received from the Grant UKM-ST-07-FRGS0028-2009.

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