Using E-learning to enhance the learning of graph transformations

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Graphical representation of functions give more meaning to algebraic expressions and is suited to a particular mathematical task. Most of the distance students in Open University Malaysia (OUM) are unable to graph functions and cannot sufficiently understand graph transformations when taught through printed learning materials. Therefore, e-learning is recommended as an aid to enhance the learning of graph transformations for various types of functions. The objectives of the e-learning are to enable students to: 1) visualize graph transformations, 2) simulate graphical representations and 3) sketch functions with guidance. The e-learning develops in a natural progression from an animated presentation to guided and advanced level exercises. The elements of animation and interactivity create an educationally stimulating environment that can enhance the learning of mathematical issues and discovery of mathematical principles. This e-learning will be tested with the students in OUM to verify its effectiveness in the learning of functions and graph transformations.

Introduction

Learning mathematics via distance education

In the distance and tutorial-web based system adopted by the Open University of Malaysia (OUM), students were supplied with a paper based learning module as the material for self-learning. They can clarify their doubts with their tutor in the tutorial session or in online discussion session. Nonetheless they have to learn independently in this distance educational system where no lecture is provided. In the learning of mathematics, they first encounter, revise, understand and practice a particular mathematical concept by themselves. Therefore, the utilization of technologies should be emphasized in this individually and technologically oriented environment. Many researchers investigated the technology usefulness toward distance learning (Christensen et al., 2001; Albrechtsen et al., 2001; Perez & Foshay, 2002) and contributed suggestions to the development of online mathematics courses (Taylor & Mohr, 2001). The effect of technology is considered in the learning of mathematics (Kelly et al., 1993), either in an algebra course for at-risk students (Pugalee, 2001) or in the learning of geometry (Stenglein, 1997) to enhance students' geometric reasoning (Battista, 2002).

Learning of functions

Basically, this research is focused on the learning of functions for the students in OUM. These students have to learn the definitions and graphical representations for various types of function, including polynomial functions (linear, quadratic and cubic), advanced functions (piecewise and rational) and special functions (exponential and logarithm). Most of the students from OUM are shocked and confused when they first encounter the many types of function at the same time. Most of them are unable to understand graph transformations and failed to sketch graphs after transformations. The worst part is they believe that more functions will appear after graph transformations. This may be due to uncontrollable visualization (Aspinwall & Shaw, 2002).

Graphing and analyzing graphical data are important for students before embarking on more complex numerical techniques (Emmons & Blubaugh, 1999). This enables students to connect algebraic equations, graphics and numerical values of the shifted functions (Vonder, 1996) and will help students to understand multi representations for mathematical concepts and to select and apply a representation that is suited to a particular mathematical task (Aspinwall & Shaw, 2002). For example, functions defined by algebraic expressions represented symbolically are easier to manipulate and analyze. Meanwhile, a graphical representation of function makes the algebraic symbol significant (Buck, 2000), accessible and can be used in a meaningful way (Dyke, 2003).

E-learning and graph transformations

Traditionally, in a face to face lecture, a lecturer will sketch and 'animate' the graph on the whiteboard so that students can visualize the graph's transformation. To improve their retention, students are required to sketch and animate the graphs on their exercise books. Hence, students have the opportunity to personally develop meaningful concepts of function and ways of reasoning instead of memorizing properties and definitions. To create a sustainable quality learning environment for the distance learners in OUM, e-learning is strongly recommended as an aid to enhance the learning of graph transformations for various types of functions. The objectives of the e-learning are to enable students: 1) to visualize the graph transformations, 2) to simulate graphical representations, and 3) to sketch functions with guidance.

Method and process

In the development of the e-learning, several multimedia elements and the structure of the e-learning were considered to enable students to obtain some advantages through e-learning. However, the effectiveness of the e-learning will only be verified when it is used by the students in OUM in the coming semester, where the students will be provided with a printed module and e-learning for the purpose of self-learning. Interviews and questionnaires will be conducted to collect student feedback on the utilization of the e-learning. Student performance will also be observed to ensure their ability to recognize, distinguish functions and their properties and sketch graphs for these functions.

Animation and interactivity

Online learning can be an effective educational intervention depending on the context and technology used (Cradler, 2003). Animation, audio feedback, sound effects, narrated explanations and interactive tools are the types of multimedia elements that support different learning styles and benefit the students in terms of entertainment, exploration, guidance and independence (Whitnah, 1997). To achieve the objectives listed above, the elements of animation and interactivity were emphasized in the development of the e-learning. An animated pedagogical agent can optimize a computer-based multimedia environment (Atkinson, 2002; Kerrigan, 2002). It can also enhance and motivate the study of mathematical issues (Fernandez et al., 1999). Meanwhile, an interactive activity can create users' control over the environment which leads them to discover and construct mathematical principles (Moyer et al., 2002).

Structure of the e-learning

Under the topic of functions, several buttons are displayed in the platform of e-learning, which can be linked to the sections of introduction, history, mapping, functions defined by equations and graphing functions. In the section of graphing functions, students first proceed through a presentation on a particular function. This is followed with some guided exercises at different levels of difficulty. A more challenging exercise is included at the end. The reiteration of the sequence from the presentation to the exercises is applied one by one to seven types of functions: linear, quadratic, cubic, piecewise, rational, exponential and logarithm functions.

Presentation on functions and graph transformations

Firstly, a definition of function and its graph are presented. For example, a quadratic function is defined as $f(x) = ax^2 + bx + c = a(x-h)^2 + k$, $a \neq 0$, where *a*, *b* and *c* are real numbers. Then, a parabola appears at a plane. At the same time, the value of *h* and *k*, vertex (h,k) and axis $x = h$ are highlighted. This is followed with the graph's transformations. The original graph of the quadratic function is shifted up to *k* units and moves from left to right until it reaches *h* units (Figure1).

(a) $f(x) = x^2$, $a > 0$ (b) $f(x) = x^2 + k$, $a > 0$ (c) $f(x) = a(x-h)^2 + k$, $a > 0$

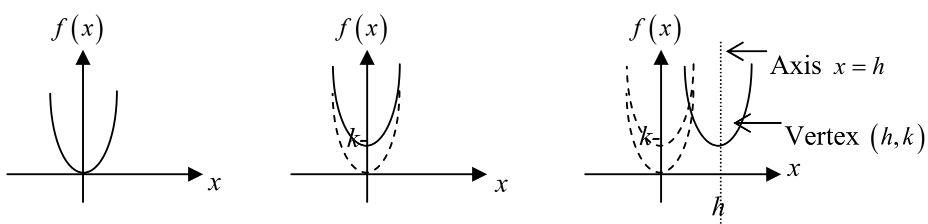

Figure 1

Guided and advanced exercises

After the presentation, an interactive guided exercise is prepared. The guided exercise is designed from a simple to complex level. At the first level, students can alter one variable in algebraic expression to observe the graph's transformation. For example, students have the flexibility to alter the value of *a* for $f(x) = ax^3$ and visualize the expansion or contraction at the graph. Afterwards, more variables can be altered, like the value of *h* and *k* in the function of $f(x) = a(x-h)^3 + k$ (Figure 2). The more challenging task is to create functions, not only writing equations or indicating the value of the vertex and axis, but also sketching graphs in advanced exercise. The interacting ability can help students to verify the algebraic expressions and graphical representations automatically.

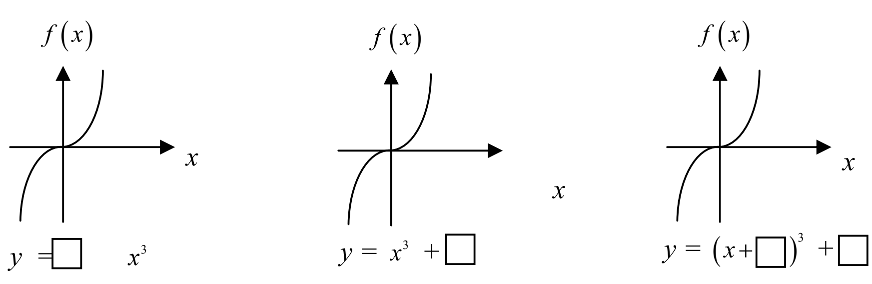

Figure 2

Advantages of e-learning

The utilization of e-learning is urged because it might create a more sustainable environment for distance learning which advantages students. Below are the expected advantages of using e-learning in the learning of functions and graph transformations.

Visualising graph transformations

To relate algebraic expressions to graphical representations, the variables in the expressions were animated and connected to the graphs during the presentation. Consider a cubic function, $f(x) = ax^3 + bx^2 + cx + d = a(x-h)^3 + k$, the variables of h and k are blinking. Then the h variable flies over to the graph of $f(x) = x^3$ while it is horizontally translating. Similarly, the k variable flies over to the graph while it is vertically translating. This will attract students' attention to the dominant variables in the algebraic expressions of the simulated graphs. It is easier to understand through animated visualization rather than explanation in the printed materials where a graph transformation is represented by dotted line. For the above function (Figure 1), several dotted lines are used to sketch

- the original graph ($f(x) = x^3$)
- the graph expansion ($f(x) = ax^3$), and
- the horizontal translation ($f(x) = a(x-h)^3$).

Finally, a graph of $f(x) = a(x-h)^3 + k$ is shown as a solid line.

Students may be confused and misunderstand that these are the graphs of various functions. Thus, the learning of graph transformations becomes complicated. In the e-learning, the original graph of the cubic function is animated in correspondence with the transformations. The graph is expanded, shifts from left to right and up to k units in a few seconds time. The animation shows a clearer picture of the transformation techniques applied on the graph. If the user is slow, they can review the animation by just clicking on the 'replay' button. Thus, the e-learning can be used based on individual progress.

Simulating graphical representations

The interactive guided exercises offer students opportunities to modify variables in algebraic expressions and visualize the corresponding simulated graphical representations of functions.

Consider the rational function, $f(x) = \frac{1}{x}$. The expansion and contradiction of the original graph are the effect of the alteration on the value of a in $f(x) = \frac{1}{ax}$. Meanwhile, the changes of the values of h and k in $g(x) = \frac{1}{a(x-h)} + k$ show the horizontal and vertical translation of the original graph.

The alteration ability provides students with more flexibility to control, predict, imagine and observe the effect of graph transformations. This advantage is very limited on printed materials. Many abstract functions are commonly seen in printed materials but only a few of the examples showing functions in multi representation. Usually, students have to sketch the graph manually if they wanted to see the effect of the transformations. This would require more time because they must draw the graph precisely. For example, the graph of $f(x) = x^2$ is hardly distinguished from the graph of $f(x) = \frac{1}{2}x^2$.

Graphing functions with guidance

The students seem to be graphing functions with guidance in the e-learning if they progress through several levels from simple to complex functions in the guided exercises. They are led step by step, from the alteration of a single variable (simulated graph's transformation) to several variables (simulated graph's transformations). For example, the original graph of a rational function, $f(x) = \frac{1}{x}$ is first vertically expanded by multiplying each ordinate value by 2, and then shifts 3 units up and move from left to the right at 1. Then, the graph is reflected in the *x*-axis to obtain the final graph for the function of $g(x) = -\frac{1}{2(x-1)} + 3$. The guided exercise is implemented in increasing sequence like a 'pyramid', which is shown as follows:

$$f(x) = \frac{1}{x}$$

$$f(x) = \frac{1}{2x}$$

$$f(x) = \frac{1}{2(x-1)}$$

$$f(x) = \frac{1}{2(x-1)} + 3$$

$$g(x) = -\frac{1}{2(x-1)} + 3$$

The students thus will accumulate the partial algebraic expressions (and the graph transformations) to attain a sophisticated expression (the final graph after transformations). After a few guided exercises, the students are expected to be ready to try a more advanced level. They can create any function by writing equations and sketching graphs. The sequence from simple to complex level exercises gives a better preparation to the students. Hence, they will understand transformations one by one and build up a strong foundation before facing a complicated expression which involves many transformations. This type of sequence is hardly shown in printed materials, where normally all the transformations are represented by a dotted line on a plane.

Conclusion

A natural progression exists from animated presentation to interactive guided exercises to advanced level exercise for each function. The students are expected to be comfortable with the graph transformations that are introduced using this progression. The advantages of this approach are that students might become familiar with the graph transformations, can connect the algebraic expression to the graphical representation, can visualize the simulating graphs and can gain proficiency in graphing and equation writing. Furthermore, learning in this environment will help students to look at functions globally to promote higher order thinking about the principles of functions and graph transformations. Hence they will aware that transformation is a tool that can be applied on various functions to gain more graphs.

In summary, the utilization of e-learning with animation and interactive elements is urged because it will help students to be independent learners and ensure the effectiveness of their learning.

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