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Integrating Constructivist approaches in e-learning to enhance mathematical self-study

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1. Introduction

1.1 Distance education

E-learning models are currently practised widely all over the world due to the rapid growth of distance and global education. Furthermore, e-learning provides high quality educational offerings (Seufert 2002) and allows a convenient and flexible learning environment without restriction of learning space, distance and time (Albrechtsen et al. 2001). Hence, distance education is widely accepted by most of the non-traditional learners (Williams et al. 2002; Leonard & Guha 2001). Open University Malaysia (OUM) is one of the distance education universities in Asia that involves in hybrid concepts for learning. With advance media and communication technologies, OUM is capable of conducting courses using a combination of self-study process (provided with printed learning material), fortnightly face to face tutorials (five tutorials, two hours each per semester) and online communication (between students-students and students-tutors).

1.2 E-learning in mathematical self-study

The various learning mediums however have not been fully utilised by the learners, especially to learn mathematics from a distance. Most of them are not comfortable with self-study and do not participate actively in online discussions. They are used to the traditional classroom learning environment where intense guidance and assistance from lecturers are provided. In distance education, self-study is actually perceived as making use of the potential in students to study on their own, where the interactive learning materials are provided at decentralised learning facilities (Davies 1998). Hence, e-learning should be used to help students in understanding and re-constructing mathematical knowledge from the paper–based materials. Nowadays, various mathematics software found in the market can be used as learning materials for mathematics (Whitnah 1997; Moyer et al. 2002).

1.3 Pedagogy approaches in e-learning

In OUM, e-learning materials are developed to complement printed-based modules. Besides technical experts such as instructional designers and programmers, mathematics lecturers also involve in the development of learning materials. Both of the mathematical content and pedagogy approaches are taken into account in designing learning materials. This study is conducted to seek appropriate approaches that can be integrated into mathematical e-learning. Specifically, the objectives of the study are to identify 1) pedagogical approaches employed by tutor for overcoming students' mathematical difficulties, and 2) multimedia elements for supporting the identified pedagogical approaches in e-learning. This study provides effective e-learning scheme which is consistent with the factors affecting mathematical self-study such as students' needs and difficulties. The aim is to encourage students to be independence in self-study.

2. Experimental design

Subjects

The subject of the study comprised fifty students who were taking Fundamental Mathematics course in OUM. They are required to pass this course before further to other courses in their fields such as science, education, information technology, business and engineering. They are selected from two classes in a learning centre located in Klang Valley. They are adult students aged from early thirty to late forty. All of them are working full time. Their gender is equally distributed. Their race and religion are not considered in this study.

Background

The syllabus of Fundamental Mathematics course covers the contents of algebra, functions and geometry. Students attended five tutorials in a semester and only several topics or parts of the contents are discussed in the tutorial due to time constraint. Tutors might explain or focus only on mathematical concepts or procedures questioned by the students. In other circumstances, tutors might only highlight the main concepts in the printed module supplied to the students.

Procedures

Firstly, the understanding of the students on the particular topics were examined either in verbal or non-verbal form. Students' difficulties in mathematics and the approaches or strategies employed by the tutors were observed. At the end of the tutorial, a quiz was conducted. Students' performances in the beginning of the tutorial and quiz were compared. Based on the findings, students were interviewed to identify methods to improve students' understanding on the topics discussed. The qualitative methods were repeatedly applied into five tutorials. Thus, the data was gathered gradually in one academic semester which approximately took three months time.

3. Results

Before tutorials started, students are requested to study on the provided printed module. However, most of them claimed that it is difficult to understand mathematical concepts and procedures properly on their own. From the observation, many mistakes made by the students are not really realized by them. These difficulties have affected the subsequent understanding on further contents and can only be corrected with the help of the tutor.

Algebra-exponent and logarithm properties

In examining students' comprehension on exponent and logarithm properties, 44% of them knew the principles of exponent when simplifying $a^2 \cdot a^3 = a^{2+3}$ and $\frac{a^4}{a^3} = a^{4-3}$, but most of them made mistakes by solving $2^3 \cdot 2^0 = 6 \times 0$ or $(-3)^2 = -9$ or $\frac{x^4 \cdot y^2}{x^3} = x^{4-3} \cdot y^{2-3}$. They only realised their mistakes when the tutor highlighted $2^3 = \underbrace{2 \times 2 \times 2}_{3 \text{ times}}$, $2^0 = 1$, $(-3)^2 = (-3)(-3)$ and explained that the principles can be used only to exponent with the same base. To improve the students' understanding, the tutor explained exponent interpretation by using concrete objects, followed with the arithmetic and only then abstract unknown, such as $a^n = \underbrace{a \times a \times ... \times a}_{n \text{ times}}$. In solving an exponential equation in the quiz, 82% of them succeeded to get an accurate value of unknown.

Functions-graphical representations

Although the definitions and graphical representation of various types of functions were displayed clearly in the module, more than 70% of the students still failed to match the given algebraic expressions ($f(x) = (x-2)^3$, $g(x) = -x^2 + 5$ and $h(x) = \frac{1}{x+3} - 5$) to the shown graphics in the initial test. The more functions they encountered in the module, the weaker their retention would be. To overcome this problem, the tutor first sketched the original graphic of a function (e.g. $f(x)=x^3$) on the whiteboard. Then, various graph transformations: vertical ($f(x)=x^3+5$) and horizontal ($f(x)=(x-2)^3$) translations, reflection ($f(x)=-x^3$), vertical expansion ($f(x)=2x^3$) and contraction ($f(x)=\frac{1}{2}x^3$) were shown in different colors. After this, students were given chances to apply graph transformations on the other functions too. The ability of graph transformation is considered as tool for them to have higher flexibility in mathematical problem solving. In the quiz at the end of the tutorial, they were able to sketch the inverse of $\ln(x-2)$ although some of them might not find the inverse in numerical form.

Trigonometry-angle direction

Students were asked to explain the meaning of positive and negative angle when the third tutorial had just started. More than half of them failed to show the rotation of $390°$ and $-270°$. Nonetheless, the following question and answer session between tutor-students had improved students' comprehension on angle direction.

Tutor : What is positive rotation?
Student : Anti-clockwise.
Tutor : Then, how about negative rotation?
Student : Clockwise.
Tutor : How many degrees are rotated from a starting point and back to that point?
Student : $360°$.
Tutor : How many degrees have to carry on for the rotation of $390°$?
Student : $30°$.

With the right concept of positive and negative angle, 92% of the students successfully determined the sign of $\sin 280°$ and $\tan 190°$ in the quiz.

4. Constructivist learning approaches

Previously, mathematics is viewed as a body of discrete, hierarchical, sequential and fixed knowledge. In the tradition instruction, teaching of mathematics occurs by telling students facts, skills, tactics, algorithms and heuristics (Grant 1998; Schoenfeld 1983). In recent years, teachers are called to support students' effort to understand a coherent, well-articulated curriculum in the student-centre learning process. This is a new learning philosophy – constructivism that become the main focus of mathematics reformation (NCTM 2000). Students are expected to construct knowledge through an active, conscious and reflective system (Clements 1997). Apparently, in this study, tutors help students to gain meaningful mathematical knowledge using the following constructivist approaches.

From concrete to abstract

For most of the students, the exponent properties are like a meaningless symbol manipulation. It became difficult to apply exponent properties flexibly in any condition such as the quotient of polynomial, numerical or unknown representations. Hence, tutor sets up a continuum of situations for the students from the manipulation of actual materials to pictures or tallies, and then in numerical ($2^3=\underbrace{2\times2\times2}_{3\text{ times}}$) and unknown form ($a^n=\underbrace{a\times a\times\ldots\times a}_{n\text{ times}}$). When the students do not use manipulative in a routine manner, they were actively constructing their knowledge. Gradually, their understanding increased from concrete to abstract, synonyms from incomplete to complete knowledge and from unsystematic to systematic thinking.

Guided discovery
Discovery through only receiving information from the module might be meaningless for the students. With the opportunity to visualize static graphic dynamically, students were able to think about the relationship between algebraic and graphical representations. When the students actively worked on various graph transformations for several types of functions, they might have thought about the role of translation, reflection, expansion and contraction on the graphs. In this way, students were allowed to build their thinking consciously and reflectively on the complex and abstract information they obtained from the module. Hence, graph transformation became a flexible tool for them, which can be applied in any function. However, the tutor's assistance is needed in the whole process, where the guided discovery process (Clements 1997) is emphasized in excogitating and constructing significant ideas.

Questions-answer session
Probing questions is one of the powerful learning strategies that encourage the learners to think critically and mathematically to develop basic analytical and computational skills (Wimer et al. 2001; Burns 2003). In the study, students were questioned on positive and negative rotation, which encouraged them to think, expand and reach the understanding of the complex rotation of $390°$ and $-270°$. Relate "anti-clockwise" to the positive rotation allows the students to reflect the situation of the negative angle as "clockwise". With the understanding of the direction of rotation, only then the students were required to interpret what they perceive on $360°$ through the observation. The questions were probing in this flow because the tutor can foresee the goal that can be achieved with this focus. Hence, from the start of recalling previous knowledge, connecting the known information with the incoming information until constructing new knowledge, students finally determined the sign of $\sin 280°$ and $\tan 190°$ successfully.

5. Multimedia elements in e-learning

Accommodating appropriate learning theories into electronic-based learning material is useful as ways of structuring learning environment in technology-enabled education. In mathematics for self-study, the constructivist framework is promoted to support activity, exploration and creation, which could assist students in constructing their own knowledge. In addition, the constructivist approach is likely tutors' favourite in helping students overcome their mathematical difficulties. Yet, rich constructivist mathematical experiences can only be obtained by self-regulated learners when self-study is complimented by utilizing technology in e-learning. (Pape & Smith 2002). Technology fits well with constructivism through animated objects, interactive tools, rich simulated worlds, narrated explanations and hypertext, which are types of multimedia elements that make sense in the constructivist paradigm (Whitnah 1997; Berg 2002). Several multimedia tools such as animation, interaction and simulation can be emphasized in the development of e-learning that consistent with the constructivist approaches.

A simulating virtual lab on object manipulation could be set up to enhance the strategy of "from concrete to abstract". The interactivity capability offers opportunity for learners to manipulate animated 3-D object as in the real condition, then to control pictures or tallies and numerical values, in a progression.

The animated presentation on graph transformation of various functions could show the relationship between algebraic and graphical representations and exhibit a clear picture of the transformation technique applied on the graph. Students may modify variables in algebraic expressions and visualize the corresponding simulated graphics in the interactive activity. Hence, this guided discovery method may encourage learners to visualize, connect, predict, imagine and explore the effect of graph transformations.

An interactive questions-answers situation could be created in e-learning which may as real as the communication between tutor and students. The questions will appear in both text and sound forms to attract students' attention. More than one answer with the similar meaning will be accepted for one question. If students are unable to answer correctly, an appropriate feedback will be given to lead students to reach the accurate answer.

6. Conclusion

Self-study is one of the components in modern distance education. Self-study in mathematics became passive when the students only study paper-based module. Their progresses in mathematics were affected by various unrealizable and uncontrollable mathematical difficulties. Their understanding can be improved with the guidance of tutors using constructivist approaches such as "from concrete to abstract", "guided discovery" and "probing questions". Nevertheless, self-study requires self-monitoring and self-reflection on outcomes rather than depending on tutors' guidance. Hence, constructivist approaches should be integrated in e-learning to enhance self-study in mathematics. This is necessary because the success of educational technology will be determined by applying effective learning theories (Berg 2000). Practising self-study in this process will lead the individual to the active exploration of facts and relationships and construction of significant mathematical concepts. Meanwhile, active engagement needs to be accompanied by advanced multimedia tools such as animation, interactivity and simulation. Only with advance technologies, students could visualize animated presentations, simulate object manipulation in virtual lab, and monitor variables in interactive activities. In summary, self-study could be effective for students to learn mathematics from a distance if they use constructivist and multimedia-based e-learning material as an addition to the paper-based module.

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